J. Fluid Mech. (1974), *vol.* 66, *part* 3, *pp.* 541–551
Printed in Great Britain

Free convection effects on steady MHD flow past a vertical porous plate

By V. M. SOUNDALGEKAR
Department of Mechanical Engineering and Engineering Production,
University of Wales Institute of Science and Technology, Cardiff

(Received 14 March 1974)

An analysis of two-dimensional steady flow of an incompressible, viscous, electrically conducting fluid past an infinite vertical porous plate is carried out under the following assumptions: (i) that the suction velocity normal to the plate is constant, (ii) that the plate temperature is constant, (iii) that the difference between the temperatures of the plate and the free stream is moderately large, causing free convection currents, (iv) that the transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible.

Approximate solutions to the coupled nonlinear equations governing the steady velocity and temperature are derived. They are shown graphically. During the course of discussion, the effects of positive and negative G (the Grashof number: $G > 0$ implies cooling of the plate, $G < 0$ heating of the plate), of P (the Prandtl number), of positive and negative E (the Eckert number) and of M (the magnetic field parameter) are presented quantitatively.

1. Introduction

In the theory of combined free and forced convection involving steady vertical flow past a hot vertical plate, an interesting question is the following. If the difference $T'_w - T'_\infty$ between the plate temperature T'_w and the free-stream temperature T'_∞ is appreciable, which causes free convection currents to flow in the boundary layer, then how is the flow field near the porous plate, with constant suction, affected by these free convection currents? This physical situation is important in technological fields. Hence, Soundalgekar (1974) attempted to solve such a problem by assuming that free convection currents were present in the boundary layer, but the flow was very slow and hence the viscous dissipative effects were negligible. This problem, governed by coupled linear equations, was completely solved by Soundalgekar and it was observed that the temperature field is in no way affected by the free convection currents. Such a situation is realistic only for very slow motion. However, in a large number of cases, the subsonic flow is not very slow. Heat due to viscous dissipation is present in many physical situations, for example, in a strong gravitational field. It was observed by Gebhart (1962) that viscous dissipative heat in natural convection is also important under the above-stated conditions. Hence, it is more appropriate to study the effects of free convection currents on the steady motion when heat due to viscous dissipation is present. An attempt in this direction was made by Soundalgekar (1973), who observed that the steady flow is considerably affected by the values of the Grashof number G, the Eckert number E and the Prandtl number P.

Now, in recent years, the effects of a transversely applied magnetic field on the flow of an electrically conducting viscous fluid have been discussed widely owing to applications in technological fields. Hence, it is the object of the present paper to analyse the effects of a magnetic field on steady forced and free convective flow past an infinite plate in the presence of suction. In § 2, the mathematical analysis for the hydromagnetic flow is presented, without taking into account the induced magnetic field. This is a valid assumption for small magnetic Reynolds number. Approximate solutions to the coupled nonlinear equations are derived for the steady flow. The velocity, temperature, skin friction and the rate of heat transfer are shown graphically followed by a quantitative discussion. The important results are summarized at the end.

2. Mathematical analysis

A steady two-dimensional vertical flow of electrically conducting, incompressible, viscous fluid past an infinite vertical porous plate with constant suction is assumed. A magnetic field of uniform strength is assumed to be applied transversely to the direction of the flow. The magnetic Reynolds number of the flow is assumed to be small and hence the induced magnetic field is neglected. If the x axis is taken in the vertical direction along the plate and the y axis is chosen normal to the plate, then under the usual Boussinesq approximation, the steady forced and free convection flow is governed by the following equations in non-dimensional form:

$$\frac{d^2u}{dy^2}+\frac{du}{dy}+M(1-u)+G\theta = 0, \tag{1}$$

$$\frac{d^2\theta}{dy^2}+P\frac{d\theta}{dy}+PE\left(\frac{du}{dy}\right)^2 = 0. \tag{2}$$

Here, the non-dimensional quantities are defined as follows:

$$\left.\begin{aligned} y &= \frac{y'v_0}{\nu}, \quad u = \frac{u'}{U_0}, \quad \theta = \frac{T'-T'_\infty}{T'_w-T'_\infty}, \quad G = \frac{\nu g\beta(T'_w-T'_\infty)}{U_0 v_0^2}, \\ P &= \frac{\mu c_p}{k}, \quad M = \frac{\sigma\beta_0^2\nu}{\rho' v_0^2}, \quad E = \frac{U_0^2}{c_p(T'_w-T'_\infty)}. \end{aligned}\right\} \tag{3}$$

Here U_0 is the free-stream velocity, M the magnetic field parameter and v_0 is the constant suction velocity. All other physical quantities have their usual meaning.

The boundary conditions are

$$\left.\begin{aligned} u &= 0, \quad \theta = 1 \quad \text{at} \quad y = 0, \\ u &= 1, \quad \theta = 0 \quad \text{as} \quad y \to \infty. \end{aligned}\right\} \tag{4}$$

To solve the coupled nonlinear equations (1) and (2) under the boundary conditions (4), we assume that

$$u = u_0+Eu_1, \quad \theta = \theta_0+E\theta_1, \tag{5}$$

where $E < 1$ for all incompressible fluids.

Substituting (5) in (1) and (2), equating the coefficients of different powers of E and neglecting those of E^2, we obtained the following solutions for the velocity and the temperature:

$$
\begin{aligned}
u = {} & 1+\left(\frac{G}{P^2-P-M}-1\right)e^{-my}-\frac{G}{P^2-P-M}e^{-Py} \\
& +GE\left[\left\{\frac{mP[G/(P^2-P-M)-1]^2}{2(P^2-P-M)(2m-P)}-\frac{2P^2[G/(P^2-P-M)-1]}{m(m+P)(P^2-P-M)^2}+\frac{PG}{2(P^2-P-M)^2}\right\}\right. \\
& \times(e^{-my}-e^{-Py})-\frac{mP[G/(P^2-P-M)^{-1}]^2}{2(2m-P)(4m^2-2m-M)}(e^{-my}-e^{-2my}) \\
& +\frac{2GP^2[G/(P^2-P-M)-1]}{m(m+P)(P^2-P-M)[(P+m)^2-(P+m)-M]}(e^{-my}-e^{-(m+P)y}) \\
& \left.-\frac{PG^2}{2(P^2-P-M)^2(4P^2-2P-M)}(e^{-my}-e^{-2Py})\right], \qquad (6)
\end{aligned}
$$

$$
\begin{aligned}
\theta(y) = {} & e^{Py}+E\left[\frac{mP[G/(P^2-P-M)-1]^2}{2(2m-P)}(e^{-Py}-e^{-2my})\right. \\
& \left.-\frac{2P^2G[G/(P^2-P-M)-1]}{m(m+P)(P^2-P-M)}(e^{-Py}-e^{-(m+P)y})+\frac{PG^2}{2(P^2-P-M)}(e^{-Py}-e^{-2Py})\right], \qquad (7)
\end{aligned}
$$

where $m = \frac{1}{2}[1+(1+4M)^{\frac{1}{2}}]$.

For $M = 0$, these equations for u and θ reduce to those obtained by Soundalgekar (1973). From (6) and (7), the numerical values of u and θ are calculated for different values of G, E, P and M. Now, all real values of G, the Grashof number, are taken into account. This is because, from the physical point of view, the value of G indicates the state of the plate. For the value of $G \equiv \nu g\beta(T'_w - T'_\infty)/U_0 v_0^2$ depends upon $T'_w - T'_\infty$, which can take positive, zero or negative values dependng upon the plate temperature. If $T'_w - T'_\infty < 0$, then the plate temperature is less than the free-stream temperature and hence the free convection currents flow towards the plate and the plate is heated from the externally supplied heat energy. Hence $G < 0$ corresponds to an externally heated plate. Similarly, $G > 0$ corresponds to external cooling of the plate and $G = 0$ then corresponds to the absence of free convection currents. As E also is a function of $T'_w - T'_\infty$, the sign of E is the same as that of G, though both cases physically correspond to the addition of viscous dissipative heat. The values of the Prandtl number are chosen to represent mercury ($P = 0{\cdot}025$) and air ($P = 0{\cdot}71$). Air is chosen because it is weakly electrically conducting under certain circumstances. The values of the Eckert number E are also very small for an incompressible fluid and hence they are chosen as 0·01 and 0·02. The values of M are chosen arbitrarily.

The velocity profiles are shown in figures 1–4.

Figure 1 shows the velocity for mercury in the case when the plate is being cooled by the free convection currents. The presence of the free convection currents causes the velocity to rise. Greater heat addition by viscous dissipation or increased cooling of the plate also results in a rise in the velocity. For possible experimental verification, the variation of the velocity is expressed in terms of the percentage change. We observe from figure 1 that at $y = 0{\cdot}2$, when G is increased from 5 to 10, there is a rise of 69 % in the velocity when $M = 2$ and

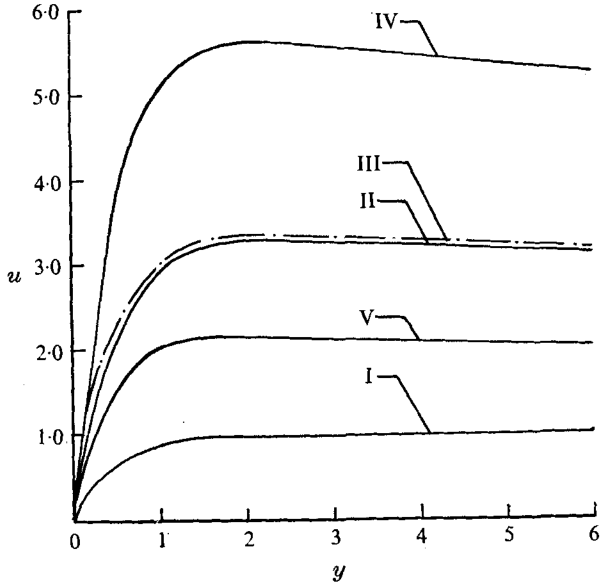

FIGURE 1. Velocity profiles for $P = 0{\cdot}025$ (mercury).

	I	II	III	IV	V
M	2	2	2	2	4
G	0	5	5	10	5
E	0	0·01	0·02	0·01	0·01

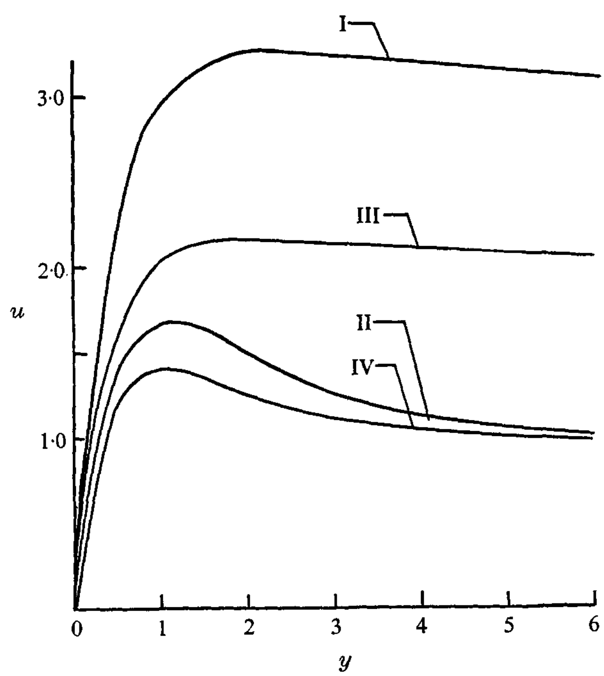

FIGURE 2. Velocity profiles for $G = 5$, $E = 0{\cdot}01$.

	I	II	III	IV
M	0	2	4	4
P	0·025	0·71	0·025	0·71

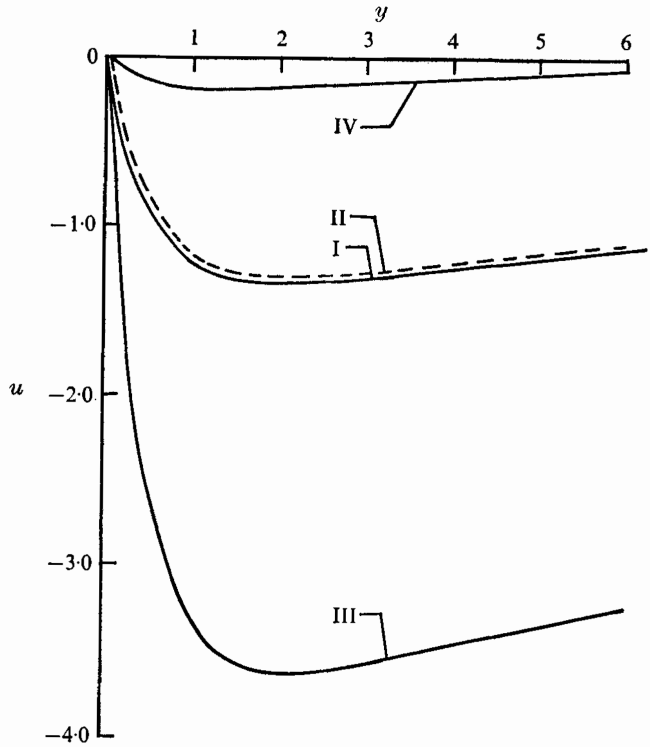

FIGURE 3. Velocity profiles for $P = 0{\cdot}025$ (mercury).

	I	II	III	IV
M	2	2	2	4
G	−5	−5	−10	−5
E	−0·01	−0·02	−0·01	−0·01

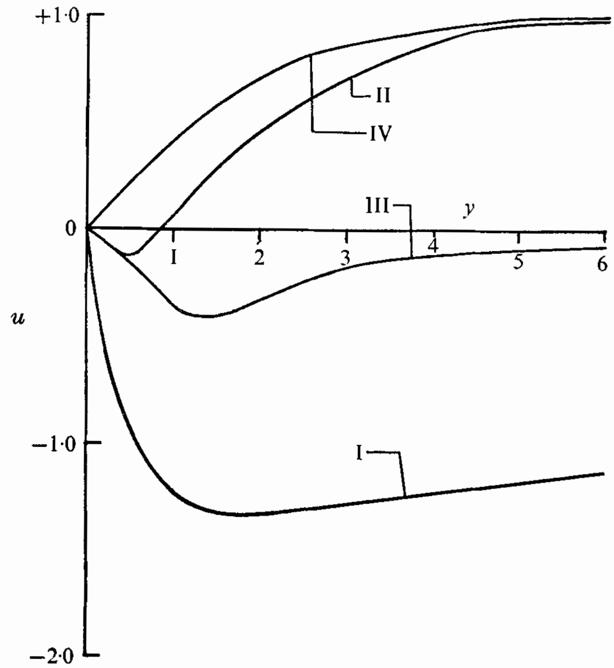

FIGURE 4. Velocity profiles for $G = -5$, $E = -0{\cdot}01$.

	I	II	III	IV
M	2	2	4	4
P	0·025	0·71	0·025	0·71

$E = 0{\cdot}01$. But for $y = 0{\cdot}2$, $G = 5$ and $E = 0{\cdot}01$, there is a fall of 33 % in the velocity when M is increased from 2 to 4. The effect of the change in the Prandtl number is shown on figure 2. We conclude that an increase in P leads to a decrease in the velocity. We also conclude from these two figures that, when the plate is being cooled by the free convection currents, the velocity is positive and hence reverse flow does not occur in this case.

The velocity profiles for the case when the plate is being heated by the free convection currents are shown in figures 3 and 4. We observe from these figures that the velocity is negative for low Prandtl number fluids. Hence, there is reverse flow in fluids with small Prandtl number. However, in the case of air, the separation may be avoided by the application of a magnetic field of high intensity. Greater heat addition by viscous dissipation causes a rise in the velocity. But increased heating of the plate makes the velocity decrease. Hence, the tendency to separation is increased by increased heating of the plate by the free convection currents.

Thus we conclude here that reversed MHD flow may be avoided by increased cooling of the plate or by an increase in M. For possible experimental verification of the effect of heating the plate, we state here the percentage fall in the value of the velocity. At $y = 0{\cdot}2$, when G is increased from 5 to 10 by increased heating of the plate, there is a fall of 200 % in the value of the velocity for $M = 2$, $E = 0{\cdot}01$ and $P = 0{\cdot}025$.

The temperature profiles for the case when the plate is being cooled by the free convection currents are shown in figure 5. For mercury, the free convection currents cause a rise in the temperature. Greater heat addition by viscous dissipation or increased cooling of the plate causes an increase in the temperature of mercury whereas an increase in M leads to a decrease in the temperature of mercury. As compared with mercury, the temperature of air decreases rather rapidly. In figure 6, the temperatures of mercury and air are shown for the case when the plate is being heated by the free convection currents. We observe from this figure that an increase in the magnetic field intensity gives a rise in the temperature. However, greater heat addition due to viscous dissipation or increased heating of the plate yields a fall in the temperature.

Knowing the velocity field, we can now calculate the skin friction, which is given by

$$\tau' = \mu[\partial u'/\partial y']_{y=0} \tag{8}$$

and, in virtue of (3), reduces to the following non-dimensional form:

$$\tau = \tau'/\rho' U_0 v_0 = [\partial u/\partial y]_{y=0}. \tag{9}$$

Hence, from (6) and (9), we obtain

$$\begin{aligned}\tau = {} & \frac{GP}{P^2-P-M} - m\left(\frac{G}{P^2-P-M}-1\right) + GE\left[\frac{P-m}{P^2-P-M}\left\{\frac{mP}{2(2m-P)}\right.\right.\\ & \times\left(\frac{G}{P^2-P-M}-1\right)^2 - \frac{2P^2[G/(P^2-P-M)-1]}{m(m+P)(P^2-P-M)} + \left.\frac{PG}{2(P^2-P-M)}\right\}\\ & - \frac{m^2P[G/(P^2-P-M)-1]^2}{2(2m-P)(4m^2-2m-P)}\\ & + \left.\frac{2GP^3[G/(P^2-P-M)-1]}{m(m+P)(P^2-P-M)[(P+m)^2-(P+m)-M]}\right].\end{aligned} \tag{10}$$

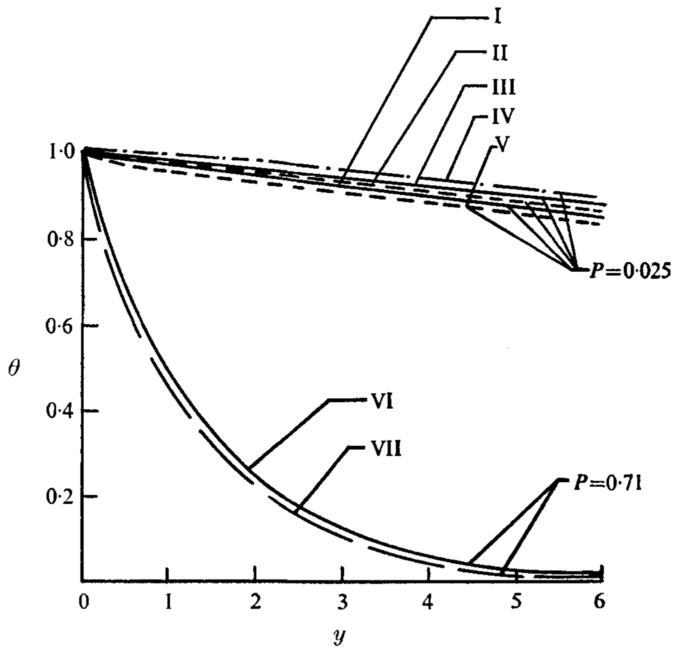

FIGURE 5. Temperature profiles.

	I	II	III	IV	V	VI	VII
M	2	2	2	2	4	2	4
G	0	5	5	10	5	5	5
E	0·01	0·01	0·02	0·01	0·01	0·01	0·01

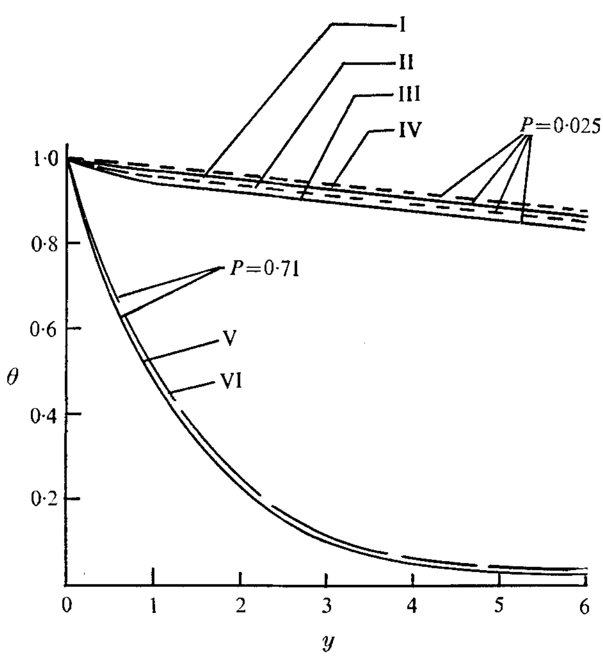

FIGURE 6. Temperature profiles.

	I	II	III	IV	V	VI
M	2	2	2	4	2	4
G	−5	−5	−10	−5	−5	−5
E	−0·01	−0·02	−0·01	−0·01	−0·01	−0·01

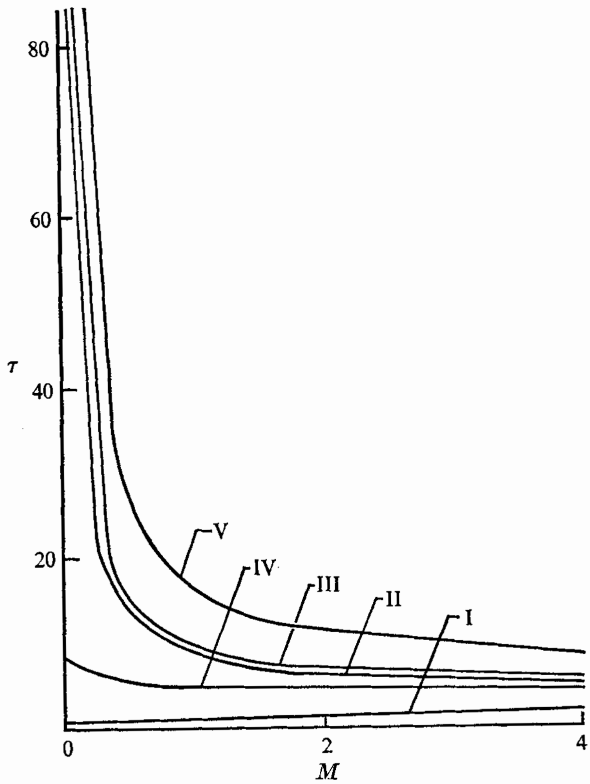

FIGURE 7. Skin friction.

	I	II	III	IV	V
G	0	5	5	5	10
E		0·01	0·02	0·01	0·01
P		0·025	0·025	0·025	0·71

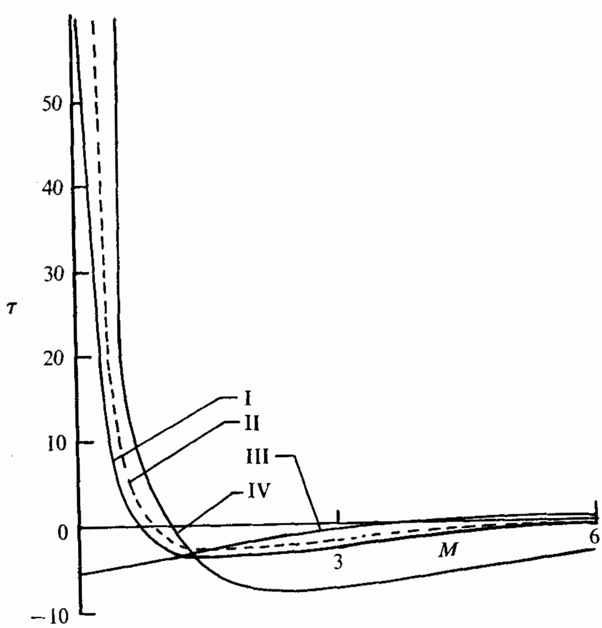

FIGURE 8. Skin friction.

	I	II	III	IV
G	−5	−5	−5	−10
E	−0·01	−0·02	−0·01	−0·01
P	0·025	0·25	0·025	0·71

τ is plotted against M in figure 7 for the case when the plate is cooled by the free convection currents. Figure 7 shows that, in the absence of free convection currents, the skin friction increases as M increases. However, in the presence of free convection currents in the case of mercury, τ decreases with increasing M. Greater cooling of the plate or greater viscous dissipation means that τ increases. Thus, for mercury, when $M = 2$ and $E = 0{\cdot}01$, there is a 100 % increase in τ when G is increased from 5 to 10, and for $M = 4$, under similar circumstances, there is 60 % rise in τ. Hence, the rate of increase of τ decreases with increasing M. τ also decreases with increasing P. In figure 8, τ is shown for the case when the plate is heated by the free convection currents. τ again decreases with increasing M. With greater heat addition by viscous dissipation, there is a rise in τ and greater heating of the plate makes τ decrease for mercury. Also, in this case, an increase in P leads to an increase in τ at large M. Thus, increased heating of the plate causes a decrease in τ and vice versa.

From the technological point of view, it is important to know the rate of heat transfer between the fluid and the plate. This is given by

$$q' = -k[\partial T'/\partial y']_{y'=0},$$

from which, by virtue of (3),

$$q = -\frac{q'\nu}{kv_0(T'_w - T'_\infty)} = \left.\frac{d\theta}{dy}\right|_{y=0}. \tag{11}$$

Hence, from (7) and (11), we get

$$q = -P + E\left[\frac{mP}{2}\left(\frac{G}{P^2-P-M}-1\right)^2 - \frac{2GP^2[G/(P^2-P-M)-1]}{(P^2-P-M)(m+P)} + \frac{G^2P^2}{2(P^2-P-M)}\right].$$

q is shown on figure 9 for $G > 0$ and on figure 10 for $G < 0$.

We observe from figure 9 that greater heat addition by viscous dissipation causes an increase in q whereas increased cooling of the plate makes q decrease at small values of M, but q is not affected by G or E at large values of M. Figure 10 shows that greater heat addition by viscous dissipation or increased heating of the plate makes q decrease at small values of M whereas, as before, q is not affected by G or E at large values of M.

3. Conclusions

(*a*) For $G > 0$, the velocity is increased by greater heat addition by viscous dissipation or increased heating of the plate and decreased by an increase in M. The velocity profiles are of non-separated type.

(*b*) For $G < 0$, the velocity profiles are of reverse type. In the case of air, separation may be avoided by the application of a strong magnetic field. Also, separation is enhanced by increased heating of the plate.

(*c*) The temperature of mercury, for $G > 0$, is increased by greater heat addition by viscous dissipation or increased cooling of the plate whereas it is decreased by an increase in M.

(*d*) For $G < 0$, the temperature of mercury is decreased by greater heat addition by viscous dissipation or increased heating of the plate and an increase M leads to an increase in the temperature.

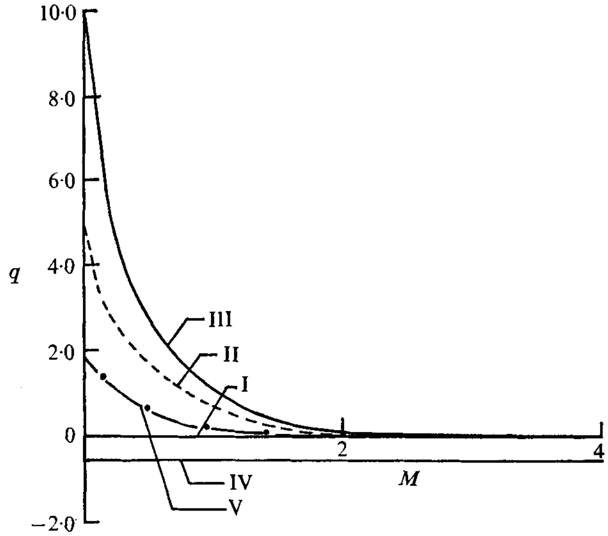

FIGURE 9. Rate of heat transfer.

	I	II	III	IV	V
G	0	5	5	5	10
E	0·01	0·01	0·02	0·01	0·01
P	0·025	0·025	0·025	0·71	0·025

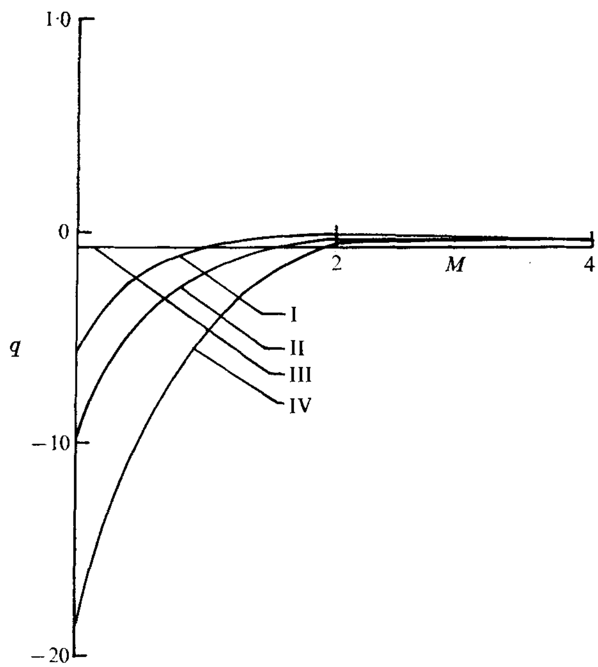

FIGURE 10. Rate of heat transfer.

	I	II	III	IV
G	−5	−5	−5	−10
E	−0·01	−0·02	−0·01	−0·01
P	0·025	0·025	0·71	0·025

(*e*) For $G > 0$, the skin friction is increased by greater heat addition by viscous dissipation or increased cooling of the plate. The rate of increase of τ decreases with increasing M.

(*f*) For $G < 0$, greater heat addition by viscous dissipation causes a rise in τ, and increased heating of the plate decreases τ for mercury. τ increases with increasing P, at large values of M.

(*g*) Greater heat addition by viscous dissipation yields a rise in the rate of heat transfer whereas greater cooling of the plate makes q decrease at small values of M. At large values of M, q is not affected by G or E.

(*h*) For $G < 0$, q is decreased by greater heat addition by viscous dissipation or greater heating of the plate at small values of M, and at large values of M, it is not affected by G or E.

I am sincerely grateful to the Leverhulme Trust for the award of a Research Fellowship and to Professor B. W. Martin for giving me all the facilities in the Department. I sincerely thank Professor Sir James Lighthill F.R.S for his critical comments on the earlier draft.

REFERENCES

GEBHART, B. 1962 *J. Fluid Mech.* **14**, 225.

SOUNDALGEKAR, V. M. 1973 *Proc. Roy. Soc.* A **333**, 25.

SOUNDALGEKAR, V. M. 1974 *Revue Roum. Phys.* (to appear).